Exercise 2D

1 a $x^2 + 6x + 1 = 0$

$x^2 + 6x = -1$

$(x + 3)^2 - 3^2 = -1$

$(x + 3)^2 = -1 + 9$

$(x + 3)^2 = 8$

$x + 3 = \pm\sqrt{8}$

$x = -3 \pm \sqrt{8}$

$x = -3 \pm \sqrt{4 \times 2}$

$x = -3 \pm 2\sqrt{2}$

$x = -3 + 2\sqrt{2}$ or $x = -3 - 2\sqrt{2}$

b $x^2 + 12x + 3 = 0$

$x^2 + 12x = -3$

$(x + 6)^2 - 6^2 = -3$

$(x + 6)^2 = -3 + 36$

$(x + 6)^2 = 33$

$x + 6 = \pm\sqrt{33}$

$x = -6 \pm \sqrt{33}$

$x = -6 + \sqrt{33}$ or $x = -6 - \sqrt{33}$

c $x^2 + 4x - 2 = 0$

$x^2 + 4x = 2$

$(x + 2)^2 - 2^2 = 2$

$(x + 2)^2 = 2 + 4$

$(x + 2)^2 = 6$

$x + 2 = \pm\sqrt{6}$

$x = -2 \pm \sqrt{6}$

$x = -2 + \sqrt{6}$ or $x = -2 - \sqrt{6}$

d $x^2 - 10x = 5$

$(x - 5)^2 - 5^2 = 5$

$(x - 5)^2 = 5 + 25$

$(x - 5)^2 = 30$

$x - 5 = \pm\sqrt{30}$

$x = 5 \pm \sqrt{30}$

$x = 5 + \sqrt{30}$ or $x = 5 - \sqrt{30}$

2 a $2x^2 + 6x - 3 = 0$

$x^2 + 3x - \frac{3}{2} = 0$

$x^2 + 3x = \frac{3}{2}$

$\left(x + \frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2 = \frac{3}{2}$

2 a $\left(x + \frac{3}{2}\right)^2 = \frac{3}{2} + \frac{9}{4}$

$\left(x + \frac{3}{2}\right)^2 = \frac{15}{4}$

$x + \frac{3}{2} = \pm\sqrt{\frac{15}{4}}$

$x = -\frac{3}{2} \pm \sqrt{\frac{15}{2}}$

$x = -\frac{3}{2} + \frac{\sqrt{15}}{2}$ or $x = -\frac{3}{2} - \frac{\sqrt{15}}{2}$

b $5x^2 + 8x - 2 = 0$

$x^2 + \frac{8}{5}x - \frac{2}{5} = 0$

$x^2 + \frac{8}{5}x = \frac{2}{5}$

$\left(x + \frac{4}{5}\right)^2 - \left(\frac{4}{5}\right)^2 = \frac{2}{5}$

$\left(x + \frac{4}{5}\right)^2 = \frac{2}{5} + \frac{16}{25}$

$\left(x + \frac{4}{5}\right)^2 = \frac{26}{25}$

$x + \frac{4}{5} = \pm\sqrt{\frac{26}{25}}$

$x = -\frac{4}{5} \pm \frac{\sqrt{26}}{5}$

$x = -\frac{4}{5} + \frac{\sqrt{26}}{5}$ or $x = -\frac{4}{5} - \frac{\sqrt{26}}{5}$

c $4x^2 - x - 8 = 0$

$x^2 - \frac{1}{4}x - 2 = 0$

$x^2 - \frac{1}{4}x = 2$

$\left(x - \frac{1}{8}\right)^2 - \left(\frac{1}{8}\right)^2 = 2$

$\left(x - \frac{1}{8}\right)^2 = 2 + \frac{1}{64}$

$\left(x - \frac{1}{8}\right)^2 = \frac{129}{64}$

2 c $x-\frac{1}{8}=\pm\sqrt{\frac{129}{64}}$

$$x=\frac{1}{8}\pm\frac{\sqrt{129}}{8}$$

$$x=\frac{1}{8}+\frac{\sqrt{129}}{8} \text{ or } x=\frac{1}{8}-\frac{\sqrt{129}}{8}$$

d $15-6x-2x^2=0$

$$-2x^2-6x+15=0$$

$$x^2+3x-\frac{15}{2}=0$$

$$x^2+3x=\frac{15}{2}$$

$$\left(x+\frac{3}{2}\right)^2-\left(\frac{3}{2}\right)^2=\frac{15}{2}$$

$$\left(x+\frac{3}{2}\right)^2=\frac{15}{2}+\frac{9}{4}$$

$$\left(x+\frac{3}{2}\right)^2=\frac{39}{4}$$

$$x+\frac{3}{2}=\pm\sqrt{\frac{39}{4}}$$

$$x=-\frac{3}{2}\pm\frac{\sqrt{39}}{2}$$

$$x=-\frac{3}{2}+\frac{\sqrt{39}}{2} \text{ or } x=-\frac{3}{2}-\frac{\sqrt{39}}{2}$$

3 a $x^2-14x+1=(x-7)^2-7^2+1$

$$=(x-7)^2-49+1$$

$$=(x-7)^2-48$$

$p=-7$ and $q=-48$

b $x^2-14x+1=0$

$$(x-7)^2-48=0$$

$$(x-7)^2=48$$

$$x-7=\pm\sqrt{48}$$

$$x=7\pm\sqrt{16\times3}$$

$$x=7\pm4\sqrt{3}$$

$r=7$ and $s=4$

4 $x^2+2bx+c=0$

$$(x+b)^2-b^2+c=0$$

$$(x+b)^2=b^2-c$$

$$x+b=\pm\sqrt{b^2-c}$$

$$x=-b\pm\sqrt{b^2-c}$$

Challenge

a $ax^2+2bx+c=0$

$$x^2+\frac{2b}{a}x+\frac{c}{a}=0$$

$$\left(x+\frac{b}{a}\right)^2-\left(\frac{b}{a}\right)^2+\frac{c}{a}=0$$

$$\left(x+\frac{b}{a}\right)^2-\frac{b^2}{a^2}+\frac{c}{a}=0$$

$$\left(x+\frac{b}{a}\right)^2=\frac{b^2}{a^2}-\frac{c}{a}$$

$$\left(x+\frac{b}{a}\right)^2=\frac{b^2-ac}{a^2}$$

$$x+\frac{b}{a}=\pm\sqrt{\frac{b^2-ac}{a^2}}$$

$$x=-\frac{b}{a}\pm\sqrt{\frac{b^2-ac}{a^2}}$$

b $ax^2+bx+c=0$

$$x^2+\frac{b}{a}x+\frac{c}{a}=0$$

$$\left(x+\frac{b}{2a}\right)^2-\left(\frac{b}{2a}\right)^2+\frac{c}{a}=0$$

$$\left(x+\frac{b}{2a}\right)^2-\frac{b^2}{4a^2}+\frac{c}{a}=0$$

$$\left(x+\frac{b}{2a}\right)^2=\frac{b^2}{4a^2}-\frac{c}{a}$$

$$\left(x+\frac{b}{2a}\right)^2=\frac{b^2-4ac}{4a^2}$$

$$x+\frac{b}{2a}=\pm\sqrt{\frac{b^2-4ac}{4a^2}}$$

$$x=-\frac{b}{2a}\pm\frac{\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$